

# Index

- American options, 178–182  
for pricing, 255–256
- American/European vanilla options, 120
- Analytic option pricing formulae, 105–113
- Annualized standard deviation, 63
- APX power exchange (APX), 192, 199
- Arithmetic progressions, 293
- Arrow–Debreu prices, 143
- Basic Linear Algebra Subprograms (BLAS), 245
- Basket options. *See* Multi-asset options
- Battery storage, 214–217
- BEGKR method, 169
- Benchmark portfolio, 234–244
- Binomial distribution, 284–289  
for large  $n$  and low probability  $p$ , 285–286  
for large number, 286–288  
normalized binomial variates, 288–289
- Binomial expansion, 294–295
- Binomial lattice, 115  
techniques, 115
- Binomial model, 171
- Bivariate cumulative normal, 174
- Black–Scholes equation, 146, 259
- Black–Scholes model, 47  
formula, 52  
Girsanov’s theorem, 53  
Greeks, 60–61, 62  
historical and implied volatility, 61–67  
inclusion of continuous dividends, 57–60
- Microsoft Excel, 67–70
- multi-asset option pricing partial differential equation, 50–52
- option pricing partial differential equation, 47–50
- probability density function, 54
- univariate cumulative normal function, 55
- Black–Scholes pricing framework, 163
- Brownian Bridge, 15–17
- Brownian motion, 1, 3  
model of asset price movements, 5  
with one source of randomness, 11–12  
properties, 3–5
- C++, 1  
classes, 245  
option pricing, 254–256  
random number class, 250–254  
risk percentiles, 256–258  
vector class, 245–250
- Call options on maximum and minimum of two assets, 174
- Central Limit Theorem, 265–266
- Confidence Intervals, 267–268
- Continuous dividends, 46–47  
inclusion, 57–60
- Contract risk distributions, 198–200
- Contract valuation, 206–209
- Correlation coefficient, 164
- Correlation matrix, 9, 83, 223–224, 272
- Covariance, 270–272  
covariance of  $n$  variables, 271–272  
three variables, 271  
two variables, 270–271
- Cox, Ross, and Rubinstein binomial lattice (CRR lattice), 118, 169
- Crank–Nicolson method, 150
- Cumulative distribution function (CDF), 22, 39, 68, 193, 282, 289
- Cumulative normal distribution function, 292–293
- Delta function, 60, 126–127, 261
- Demand side response (DSR), 2
- Dependent variables, 165
- Doubly truncated Weibull distribution, 98–102, 202–203
- Downward branching node, 139, 142–143, 145
- Dual cash-out price method, 190–191

- Early exercise, 155–159  
 Efficient frontier, 225, 226  
     with no transaction costs, 225–233  
     with transaction costs and benchmark  
         portfolio, 234–244  
 Empirical CDFs, 193, 289–290  
 European options, 173  
     call options on maximum and  
         minimum of two assets, 174  
     for pricing, 254–255  
     put options on maximum of two assets,  
         175–178  
     put options on minimum of two assets,  
         174–175  
 Excel Visual Basic code, 223  
 Exponential distribution, 34, 281–282  
 Fast response generation units, 206,  
     210  
 Financial risk distributions, 189  
 Finite difference  
     approximation, 146–150  
     lattices, 169  
     method, 160–162  
 Forward price, 76–77  
 Four-state Markov Chain, 195  
 Fubini’s theorem, 22–23, 297  
 Fully implicit method, 149–150  
 Gamma function, 60, 95, 127, 260–261  
 Gaussian Copula, 194  
 Gaussian distribution, 7, 16  
 GBP/EUR exchange rate, 34  
 GBP/USD exchange rate, 34  
 Geometric Brownian motion (GBM), 1,  
     5, 47, 115  
 Geometric progressions, 293  
 Girsanov’s theorem, 8  
 Greeks, 60–61, 125–129  
     Delta, 261  
     European call, 62  
     European put, 62  
     Gamma, 260–261  
     Rho, 263  
     Theta, 262–263  
     for Vanilla European options,  
         259–260  
     Vega, 263–264  
 Grid methods for Vanilla options, 145  
     log-transformed grids, 159–162  
     standard grids, 146–159  
     stochastic process, 145  
 Half hourly marginal system price  
     benefit, 194  
 Half hourly power price, 42–44  
     simulation, 36  
 Historic portfolio volume forecast error  
     distribution, 192–193  
 Imbalance risk, 189  
     benefit of including customer into  
         portfolio, 194–195  
     contract risk distributions, 198–200  
     contracted energy, 190–191  
     MIP, 195–197  
     risk distribution, 192–193  
     SBP and SSP, 197–198  
     single cash-out price, 191–192  
     stand-alone cost, 193–194  
 Independent, identically distributed  
     random variables (IID random  
         variables), 265, 269–270, 273  
     IID lognormal distribution, 102  
 Intraday generation, 217–221  
 Intraday power storage and demand  
     optionality, 210  
     battery storage, 214–217  
     import site with storage and solar PV,  
         212–213  
     storage connected to grid with/without  
         solar PV generation, 211–212  
     swing contract, 213–214  
 Ito product rule, 11  
     Brownian Motion with one source of  
         randomness, 11–12  
         in  $n$  dimensions, 14–15  
 Ito quotient rule, 12–14  
 Ito’s formula. *See* Ito’s lemma  
 Ito’s isometry–correlated processes,  
     25–27  
 Ito’s isometry–single process, 23–25  
 Ito’s lemma, 5–8, 297  
     for multi-asset geometric Brownian  
         motion, 9–11

- Johnson binomial lattice, 133–137  
 Johnson distribution, 84  
     option pricing formula, 84–88  
     parameter estimation, 89–93  
 Jump diffusion process, 103  
 Kalman filter, 82  
 Lattice methods, 145  
     constructing and using standard binomial lattice, 121–129  
 Johnson binomial lattice, 133–137  
 log transformed binomial lattice, 129–133  
 standard binomial lattice, 115–121  
 trinomial lattice, 137–145  
     for Vanilla options, 115  
 Least squares Monte Carlo optimization, 211–212, 213  
 Lithium ion batteries, 210  
 Load management, 35  
 Log transformed binomial lattice, 129–133  
 Log-transformed grids, 159  
     derivation of equation, 159–160  
     finite difference method, 160–162  
     *see also* Standard grids  
 Lognormal distribution, 117, 280–281  
 Longstaff Schwartz regression  
     approach, 2  
     *see also* Least squares Monte Carlo optimization  
 Marginal power price, 33  
 Market index price (MIP), 190, 195–197  
 Markov process, 3  
 Markowitz efficient frontier, 225  
 Markowitz mean–variance portfolio selection problem, 223  
 Markowitz portfolio optimization, 1  
 Martingale measure, pricing derivatives using, 45–46  
 Martingale process, 3  
 Mathematical reference  
     arithmetic and geometric progressions, 293  
     cumulative normal distribution function, 292–293  
     series expansions, 294–295  
     standard integrals, 291–292  
 Mersenne Twister uniform random number generator, 250, 252  
 Merton jump diffusion model, 102, 255  
     analytic option pricing formulae, 105–113  
     jump diffusion process, 103  
 Monte Carlo simulation, 104–105  
     parameter estimation, 113–114  
 Microsoft Excel, 67–70  
 Microsoft Excel VBA code, 1, 90–93, 223, 230, 240–242  
 Mixed integer linear programming (MILP), 211  
 Moment generating functions, 272–273  
 Monte Carlo  
     fundamental power stack model, 2  
     lattice approach, 165  
     methods, 163  
     simulation, 33, 104–105, 192, 250  
 Multi-asset  
     Black–Scholes equation, 163–164  
     derivative, 50  
     geometric Brownian motion, 1, 9–11  
 Multi-asset options, 165  
     multidimensional lattice methods, 163, 169–171  
     multidimensional Monte Carlo methods, 165–169  
     pricing partial differential equation, 50–52  
     three asset options, 183–187  
     two asset options, 171–182  
     *see also* Single asset American style options; Single asset European options  
 Multifactor forward curve model, 82–83  
 N2EX, 192  
 National Grid, 210  
 Newton’s method, 64, 66  
 Normal (Gaussian) distribution, 277–280  
 Normal cumulative distribution, 67  
 Normalized binomial variates, 133, 288–289  
 NORMDIST function, 67  
 Numéraires, 45

- Object-based programming languages, 248
- Objective Function, 1, 230, 233, 244
- One factor forward curve model, 72, 137
- exponential factor, 72
  - forward price and spot price, 76–77
  - option pricing formula, 77–80
  - spot price process, 73–76
- One-factor spot model, 70–72
- Option payoff at terminal nodes, 123–124
- Option pricing, 254–256
- American options, 255–256
  - European options, 254–255
  - formula, 77–80, 84–88
  - partial differential equation, 47–50
- Option values computation at given time instant, 151–155
- Ornstein–Uhlenbeck process, 1, 17
- mean, 18
  - variance, 19–21
- Payoff, 45, 46, 54, 123–124, 172, 213
- Poisson distribution, 282–283
- Poisson process, 21–22, 34, 102
- Poisson random number generator functions, 253
- Portfolio optimization, 224
- covariance matrix, 223–224
  - efficient frontier with no transaction costs, 225–233
  - efficient frontier with transaction costs and benchmark portfolio, 234–244
  - optimum asset allocation, 224–225
- Portfolio volume forecast error distributions, 194
- Power contracts
- imbalance risk, 189–200
  - intraday generation, 217–221
  - intraday power storage and demand optionality, 210–217
  - wind contracts, 200–209
- Power price model
- modeling wind and solar generation, 36–42
  - power stack model, 33–36
  - simulated half hourly power price, 42–44
- Power spot price, stochastic process for, 205–206
- Power stack model, 33–36
- Price European exchange options, 171
- Probability
- density function, 23, 54, 58, 275, 277
  - distribution, 15
  - measure, 53
- Problems, 27–32
- answers to, 297–313
- Put options
- on maximum of two assets, 175–178
  - on minimum of two assets, 174–175
- Put-call parity, 46–47
- Radon–Nikodym derivative, 8
- Random number class, 250–254
- Random walk, 3
- Reasonable approximation, 119
- Relative contract maturities, 83
- Renewable energy, 2
- generators, 189
- Risk percentiles, 256–258
- Simulation, 204–205
- Monte Carlo simulation, 33, 104–105, 192, 250
  - Single asset American style options
  - Grid methods for Vanilla options, 145
  - lattice methods for Vanilla options, 115–145
- see also* Multi-asset options
- Single asset European options
- Johnson distribution, 84–93
  - Merton jump diffusion model, 102–114
  - multifactor forward curve model, 82–83
  - one factor forward curve model, 72–80
  - one-factor spot model, 70–72
  - pricing derivatives using martingale measure, 45–46
  - put–call parity, 46–47
  - two-factor spot model, 81–82
- Vanilla options and Black–Scholes model, 47–70
- Weibull distribution, 93–102
- see also* Multi-asset options

- Single cash-out price method, 190–192  
 Single-factor forward curve model, 82  
 Solar generation, 35, 36  
   actual half hourly UK summer solar  
     PV generation, 41  
   actual half hourly UK winter solar PV  
     generation, 42  
   current and previous daily average UK  
     solar PV generation, 38  
   current and previous half hour UK  
     solar PV generation, 38  
   simulated half hourly UK summer  
     solar PV generation, 41  
   simulated half hourly UK winter solar  
     PV generation, 42  
 Solar PV generation  
   import site with storage and, 212–213  
   storage connected to grid  
     with/without, 211–212  
 Spot price process, 73–77  
 Standard Binomial lattice, 115  
   asset values to lattice nodes, 122–123  
   computing Greeks, 125–129  
   constructing and using, 121  
   iterate backwards through lattice,  
     124–125  
   lognormal mean, 116  
   lognormal variance, 116–121  
   option payoff at terminal nodes,  
     123–124  
   values of constants by lattice, 122  
 Standard Brownian motion, 4  
 Standard grids, 146  
   backwards iteration and early exercise,  
     155–159  
   boundary conditions, 150–151  
   computation of option values at given  
     time instant, 151–155  
   finite difference approximation,  
     146–150  
   *see also* Log-transformed grids  
 Standard integrals, 291–292  
 Standard normal distribution, 4  
 Standard statistical results, 265–273  
   Central Limit Theorem, 265–266  
   Confidence Intervals, 267–268  
   covariance, 270–272  
   covariance matrix, 272  
   law of large numbers, 265  
   moment generating functions, 272–273  
   variance, 268–270  
 Standard Template Library (STL), 245  
 Standard Weibull distribution, 93–98  
 Statistical distribution functions  
   Binomial distribution, 284–289  
   empirical CDF, 289–290  
   exponential distribution, 281–282  
   lognormal distribution, 280–281  
   normal (Gaussian) distribution,  
     277–280  
   Poisson distribution, 282–283  
   uniform distribution, 275–277  
 Stochastic integral expectation, 27  
 Stochastic integrals, 22  
   expectation of stochastic integral, 27  
   Fubini's theorem, 22–23  
   Ito's isometry–correlated processes,  
     25–27  
   Ito's isometry–single process, 23–25  
 Stochastic processes, 1, 8, 145  
   for power spot price, 205–206  
 Swing contract, 213–214  
 System buy price (SBP), 190, 197–198  
 System long, 190  
 System price, 189  
 System sell price (SSP), 190, 197–198  
 System short, 190  
 Taylor Series, 294  
 Terminal nodes, option payoff at,  
   123–124  
 Tesla, 210  
 Three asset options, 183–187  
 Tidal power, 2  
 Time varying drift and volatility, 8  
 Time-varying mean, 76  
 Trinomial lattice, 137–145  
   branching types for nodes in trinomial  
     lattice, 139  
   downward branching node, 142–143,  
     145  
   mean reverting trinomial lattice, 138  
   normal branching node, 140–141, 145  
   pricing using lattice, 144  
   upward branching node, 141–142,  
     144–145

- Two asset options, 171  
   American options, 178–182  
   European exchange options, 171–173  
   European options on maximum or minimum, 173–178
- Two dimensions, Ito product and quotient rules in, 11–14
- Two-factor spot model, 81–82
- Uniform distribution, 275–277
- Uniform grid, 156
- Univariate cumulative normal function, 55
- Value at risk (VAR), 192
- Vanilla European options, Greeks for, 259–264
- Vanilla options, 47–70
- Vanilla options, grid methods for, 145  
   log-transformed grids, 159–162  
   standard grids, 146–159  
   stochastic process, 145
- Vanilla options, lattice methods for, 115  
   constructing and using standard binomial lattice, 121–129
- Johnson binomial lattice, 133–137
- log transformed binomial lattice, 129–133
- standard binomial lattice, 115–121
- trinomial lattice, 137–145
- Variance, 268  
   of  $n$  variables, 269–270
- one variable, 268
- Ornstein–Uhlenbeck process, 19–21
- three variables, 269
- two variables, 268–269
- Vector class, 245–250
- Vega function, 60, 128–129, 263–264
- Visual Basic, 67
- Volatility, 5  
   historical, 61–64  
   implied, 61, 64–67  
   smile, 64
- Weibull distribution, 93, 202–204  
   doubly truncated Weibull distribution, 98–102  
   standard Weibull distribution, 93–98
- Wind contracts, 200  
   contract valuation, 206–209  
   simulation and calibration, 204–205  
   stochastic process for power spot price, 205–206
- Weibull distribution, 202–204
- Wind generation, 35–36  
   actual half hourly UK, 39  
   actual half hourly UK summer, 40  
   actual half hourly UK winter, 40  
   current and previous day wind generation load factors, 37  
   current and previous half hour wind generation load factors, 37  
   simulated half hourly UK, 40